# Graphical Design Method For Traveling Wave Amplifier Based On Filter Theory

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## Abstract

A graphical procedure for a straightforward Traveling Wave Amplifier design is presented. The introduction of filter theory with respect to the conventional methods allows for a sensible improvement of the input and output VSWR of the circuit. The use of normalized parameters and charts brings a process independent applicability of the method.

## Introduction

The application of Traveling Wave Amplifier (TWA) conventional design methods typically provides flat gain performance, even if the realization of low input and output VSWR remains a critical aspect. In this paper significant advances are obtained introducing a graphical, process independent TWA design procedure based on Tchebyscheff filter theory. The filter elements are defined as function of the amplifier gain, ripple and cutoff frequency. As a consequence, in contrast to the conventional approach [1,2], different values for the capacitances of the resulting filter are necessary. Since these capacitances are directly related to the parasitics of the FET, devices with appropriate gate widths can be easily realized in monolithic technology. Furthermore as the resulting formulas and charts are normalized on technological parameters, the suggested procedures can be applied directly to the specific amplifier design.

## Theory and Graphical Procedure

The classical design procedure for Traveling Wave Amplifier (fig.1) is based on the definition of the proper inductance value to form artificial transmission line with characteristic impedance:

(1) $$Z_0 = \sqrt{\frac{L}{C_{gs}}} = \sqrt{\frac{L}{C_{ds}+C_a}} = 50\Omega$$

where $C_a$ is an additional shunt capacitance.
The artificial transmission line matching degradation at high frequency and the sensitivity to the FET parasitics shown that this simple method is a not optimized one.

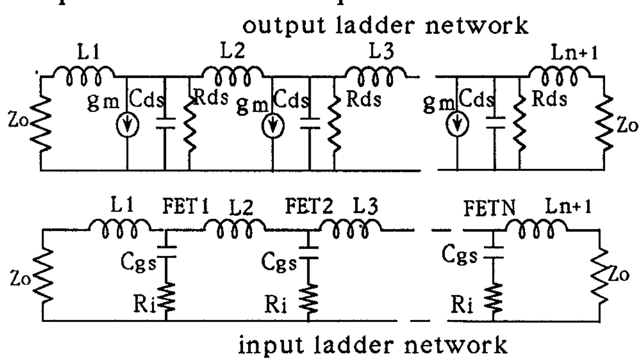
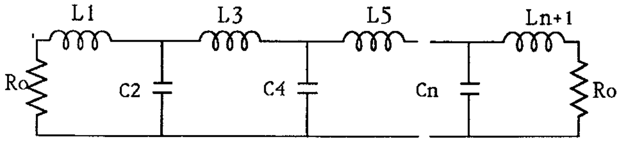



Fig1. Traveling Wave Amplifier Schematic

A different procedure based on filter theory is now proposed. The unilateral ($C_{gd}$=0), lossy equivalent model for the FET is assumed (fig.1). The input and output ladder networks are considered as identical low pass Tchebyscheff filters (fig.2) coupled by the FET transconductances.
The well known electric properties of the Tchebyscheff filter are, as a consequence, extended to the TWA performance with evident benefits in bandwidth and reflection characteristics.
The method can be simply outlined as follows. Once the 3dB cutoff frequency $f_{3dB}$ is defined as well as the number of FETs N in the amplifier, the corresponding Tchebyscheff filter lumped




$$\beta=\ln\left(\coth\frac{A_r}{17.33}\right); \qquad \gamma=\sinh\left(\frac{\beta}{2n}\right)$$

$$a_k=\sin\left[\frac{(2k-1)\pi}{2n}\right]; \qquad b_k=\gamma^2+\sin^2\left(\frac{k\pi}{n}\right) \qquad k=1,2,..,n;$$

$$g_1=\frac{2a_1}{\gamma}; \qquad g_k=\frac{4\,a_{k-1}\,a_k}{b_{k-1}\,g_{k-1}} \qquad k=2,3,...,n;$$

$$L_k=\frac{g_k R_0}{2\pi f_{fi}} \quad k=1,3,..,n+1; \qquad C_k=\frac{g_k}{2\pi f_{fi} R_0} \quad k=2,4,..,n;$$

Ar is the dB passband ripple, n is the order of the filter, $f_{fi}$ is the cutoff frequency, $R_0$ is the termination resistance

Fig.2 Tchebyscheff filter

elements are derived according to:
- the order n of the filter:

(2) $n=2N+1;$

the filter cutoff frequency $f_{fi}$:

(3) $$f_{fi}=\frac{f_{3dB}}{\cosh\left[\frac{\cosh^{-1}\sqrt{\frac{.9953}{\varepsilon^2}}}{n}\right]}$$

where $\varepsilon^2=10^{.1A_r}-1$ is the term related to the pass-band ripple $A_r$ (the optimum value $A_r$=0.01dB in the following is assumed).

According to the expression in fig.2 the values of series inductance $L_i$ and shunt capacitors $C_i$ are calculated.

The input ladder network of the TWA is generated imposing each shunt capacitor $C_i=C_{gsi}$ of the i-th FET with a properly chosen gate width $w_i$. The output ladder network, equal to the input one, is obtained by imposing $C_i=C_{dsi}+C_{ai}$, where $C_{ai}$ is an additional capacitor. It is evident that a TWA composed by FETs with different gate width results and its realizability is assured by the FET gate width scaling feature typical of the monolithic technology.

To characterize the Tchebyscheff Traveling Wave Amplifier (TTWA) in terms of low frequency gain, an equivalent TWA based on FETs with fixed gate widths is also introduced. Assuming a FET average gate width:

(4) $$\overline{w}=\frac{1}{N}\sum_{1}^{N} w_i$$

where $w_i$ is the gate width of the different FETs composing the TTWA, then the corresponding TWA based on identical FETs (each having a gate width $\overline{w}$) and consequently with the same gain parameter like the TTWA is defined.

After this assumption the low frequency gain expression from [1] based on equivalent lossy model can be effectively extended to the equivalent equal FET TWA:

(5) $$A_0=\overline{g}_m\,Z_0\,\frac{\sinh(b)\,e^{-b}}{2\sinh\left(b/N\right)}$$

where:
- $\overline{g}_m$ is the transconductance of the average FET;
- b is a lossy factor associated to the drain line expressed as [1]:

(6) $$b=\frac{N}{4}\left(\frac{Z_0}{\overline{R_{ds}}}\right)$$

$\overline{R_{ds}}$ is the $R_{ds}$ of the average FET.

An effective graphical design procedure for the Tchebyscheff based TWA is derived. The parameters typical for the FET fabrication process are introduced by scaling the elements of the equivalent circuit on the gate width w:

(7) $$k_g=\frac{g_m}{mS}\Big/\frac{w}{\mu m}; \qquad (8)\quad k_{C_{gs}}=\frac{C_{gs}}{pF}\Big/\frac{w}{\mu m}$$

(9) $$k_{R_{ds}}=\frac{R_{ds}}{\Omega}\frac{w}{\mu m}; \qquad (10)\quad k_{R_i}=\frac{R_i}{\Omega}\frac{w}{\mu m}$$

Since an average FET is considered, after simple algebra the elements of its equivalent circuit are calculated as follows:

(11) $$\overline{g}_m=\overline{C_{gs}}\,\frac{k_g}{k_{C_{gs}}}\,\text{GHz}$$

Since

(12) $$\overline{C}_{gs}=\frac{1}{N}\sum_{1}^{N} C_{gsi} \quad \text{and} \quad (13)\quad C_{gsi}=\frac{g_k}{2\pi\,f_{fi}\,Z_0}$$

and assuming:

(14) $$G=\frac{1}{N}\sum_{1}^{N} g_k$$

where $g_k$ are the filter coefficients related to the shunt capacitors in fig.2, follows:

$$\overline{C}_{gs} = \frac{G}{2\pi\, f_{fi}\, Z_0} \qquad (15)$$

so the low frequency gain $A_0$ is expressed as:

$$A_0 = \frac{k_g}{k_{C_{gs}}} \frac{G}{2\pi\, f_{fi}/GHz} \frac{\sinh(b)\, e^{-b}}{2\sinh(b/N)} \qquad (16)$$

defining

$$L(b) = \frac{\sinh(b)\, e^{-b}}{2\sinh(b/N)} \qquad (17)$$

From (6) we obtain:

$$b = \frac{NG10^3}{4\, k_{R_{ds}}\, k_{C_{gs}}\, 2\pi\, f_{fi}/GHz} \qquad (18)$$

A normalized frequency $f_N$ is also defined as:

$$f_N = \frac{f_{fi}/GHz\, k_{Cgs}\, k_{Rds}}{G\, 10^3} \qquad (19)$$

so results:

$$b = \frac{N}{8\pi f_N} \qquad (20)$$

and after simple algebra the process independent low frequency gain $A_{0N}$ is expressed as:

$$A_{0N} = \frac{A_0\, 10^3}{k_g\, k_{Rds}} = \frac{L(b)}{2\,\pi\, f_N} \qquad (21)$$

From the previously proposed theory two design charts are derived.
The factor $(f_{fi}/GHz)/G$ versus $f_{3db}$ is plotted on Chart 1 (fig.3). The process independent low frequency gain $A_{0N}$ versus $f_N$ (Equation (21)) is plotted on Chart 2 (fig. 4). The cases for N= 4, 5, 6 and 7 are considered up to 50 Ghz.
Finally the graphical procedure can be outlined as follows:
- Choose the 3dB cutoff frequency $f_{3dB}$ and the number of FET N;
- Derive from Chart 1 the correspondent value of the factor $(f_{fi}/GHz)/G$ for the given $f_{3dB}$;
- Calculate the correspondent $f_N$ from (19);
- Derive from Chart 2 the $A_{0N}$ value corresponding to $f_N$;

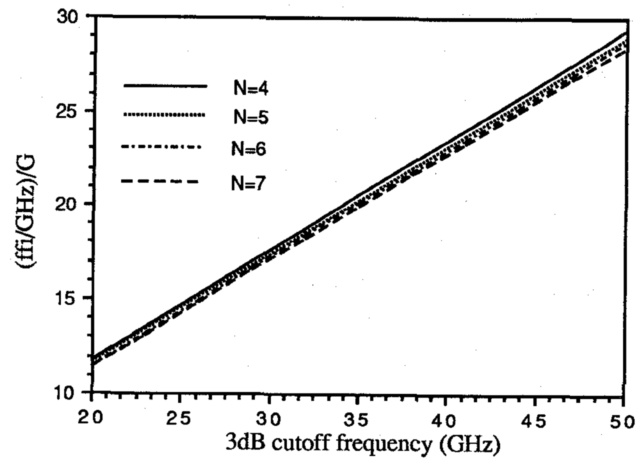


Fig.3 Chart 1

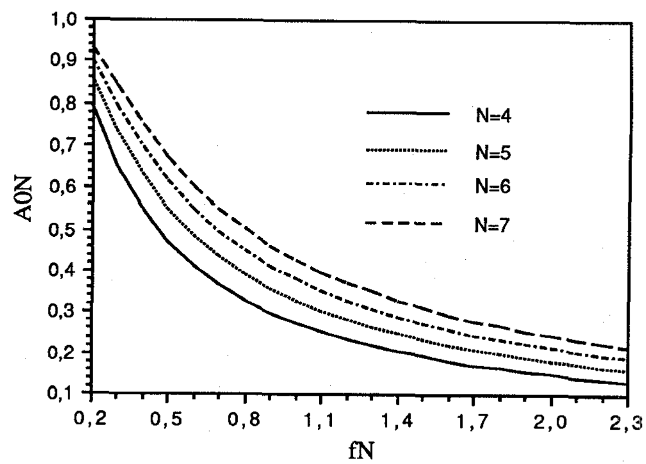


Fig.4 Chart 2

- Calculate the low frequency gain from

$$A_0 = A_{0N}\, k_g\, k_{Rds} / 10^3$$

- If the result is not satisfactory, repeat the procedure increasing the number of FET N;
- If the low frequency gain $A_0$ is satisfactory, a corresponding Tchebyscheff filter is designed applying the procedure in fig.2.
- Replace the shunt capacitors with FETs whose gate widths assure $C_{gs}$ values that are in correspondence to required filter capacitor.

## Results and Comparison

The validity of the described design technique is demonstrated by a comparison with the methods in [1] and [2], devoting particular attention to the reflection characteristics aspect. A four FETs TWA (N=4) with $f_{3dB}$=21Ghz ($f_{fi}$=19.31GHz) [2] is investigated.

The FET parameters from [1] are :

w=300 µm; $g_m$=40 mS; $C_{gs}$= .27 pF; $R_{ds}$= 300 Ω; Ri= 7 Ω,

so according to Eqns. (7)-(10):

$k_g$=.1333mS/µm ; $k_{Cgs}$=9.10$^4$pF/µm; $k_{Rds}$=90000Ωµm ; $k_{Ri}$=2100Ωµm.

Applying the graphical procedure from the two Charts the following values are derived :

$f_N$=.997 ; $A_{0N}$=.28 .

So the low frequency gain for the correspondent Tchebyscheff Traveling Wave Amplifier results (Equation (21)):

$A_0$=3.28 (10.31 dB)

The Tchebyscheff filter lumped elements values also result :

$g_1=g_4$=1.427; $g_2=g_3$=1.7125; G=1.569
$L_1=L_5$= .3354 nH; $L_2=L_4$= .743 nH; $L_5$= .785 nH;
$C_1=C_4$= .235 pF; $C_2=C_3$=.2826 pF.

A FET gate width w=261 µm assures a $C_{gs}$ values corresponding to $C_1$ and $C_4$ and w=314 µm to $C_2$ and $C_3$.
The comparison is presented displaying the gain (fig. 5), the input reflection coefficient (fig.6) and the ouput reflection coefficient (fig.7) for the amplifiers designed with the three different methods. It is immediately evident that the Tchebyscheff Traveling Wave Amplifier presents a sensitive improvement in the reflection behavior, especially in the high frequency side of the band, without affecting the gain and the ripple.

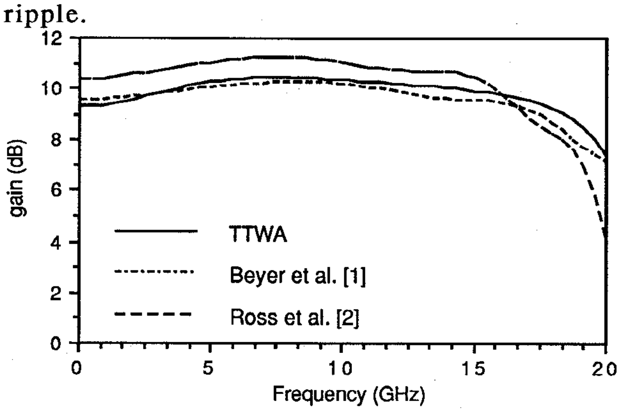


Fig.5 Gain comparison

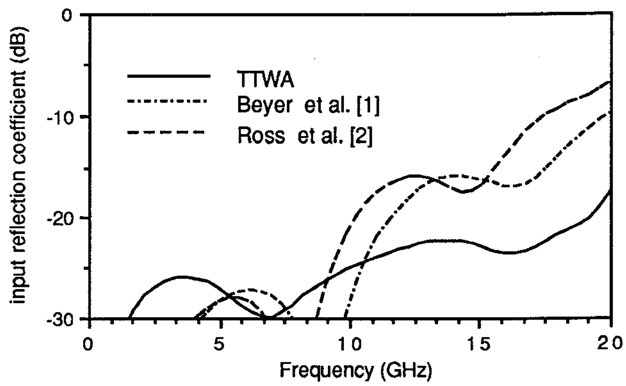


Fig.6 Input reflection coefficient comparison

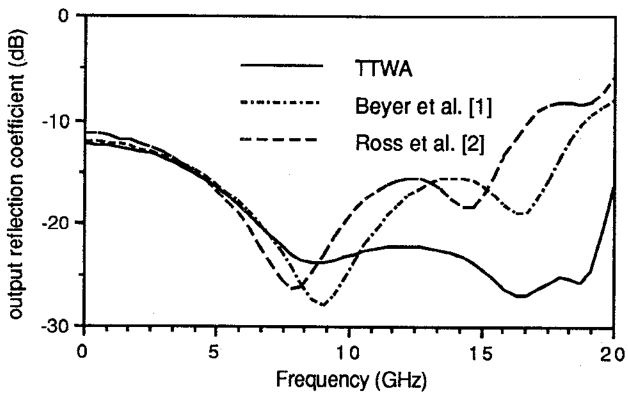


Fig.7 Output reflection coefficient comparison

## Conclusion

A straightforward graphical procedure for the design of monolithic Traveling Wave Amplifiers based on Tchebyscheff filter theory has been described. Respect to the conventional methods [1,2] a TWA with different gate widths results. The significant improvement of the Tchebyscheff TWA (TTWA) related to input and output reflection characteristics with respect to a conventional procedure becomes obvious.

## References

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